

( 3 Hours )

( Total Marks: 80 )



Please check whether you have the right question paper.

- N.B.: 1) Questions No. 1 is compulsory.  
2) Answer any three from remaining five questions.

1. a) If  $\tan \frac{x}{2} = \tan h \frac{u}{2}$ , show that  $u = \log \left[ \tan \left( \frac{\pi}{4} + \frac{x}{2} \right) \right]$ . [3]
- b) Prove that the following matrix is orthogonal & hence find  $A^{-1}$ , [3]
- $$A = \frac{1}{3} \begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$
- c) State Euler's theorem on Homogeneous function of two variables & if  $u = \frac{x+y}{x^2+y^2}$  then evaluate  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ . [3]
- d) If  $u = r^2 \cos 2\theta$ ,  $v = r^2 \sin 2\theta$ . Find  $\frac{\partial(u, v)}{\partial(r, \theta)}$ . [3]
- e) Find the  $n^{\text{th}}$  derivative of  $\cos 5x \cdot \cos 3x \cdot \cos x$ . [4]
- f) Evaluate :  $\lim_{x \rightarrow 0} \left( \frac{2x+1}{x+1} \right)^{\frac{1}{x}}$ . [4]
2. a) Solve  $x^4 - x^3 + x^2 - x + 1 = 0$ . [6]
- b) If  $y = e^{\tan^{-1} x}$ . Prove that  $(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$ . [6]
- c) Examine the function  $f(x, y) = xy(3-x-y)$  for extremes values & also find maximum and minimum values of  $f(x, y)$ . [8]

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3. a) Investigate for what values of  $\lambda$  &  $\mu$  the equations  $x+y+z=6$ ; [6]  
 $x+2y+3z=10$ ;  $x+2y+\lambda z=\mu$  have

- i) no solution,  
 ii) a unique solution,  
 iii) infinite no. of solutions.

b) If  $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ , show that  $x^2 \frac{\partial u}{\partial y} + y^2 \frac{\partial u}{\partial x} + z^2 \frac{\partial u}{\partial z} = 0$ . [6]

c) Prove that  $\log\left(\frac{a+ib}{a-ib}\right) = 2i \tan^{-1}\left(\frac{b}{a}\right)$  & [8]

$$\cos\left[i \log\left(\frac{a+ib}{a-ib}\right)\right] = \frac{a^2 - b^2}{a^2 + b^2}.$$

4. a) If  $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$ , Prove that [6]

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{-\sin u \cos 2u}{4 \cos^3 u}.$$

b) Using encoding matrix  $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ; encode & decode the message [6]

‘ALL IS WELL’.

c) Solve the following equations by Gauss Seidal method : [8]

$$10x_1 + x_2 + x_3 = 12$$

$$2x_1 + 10x_2 + x_3 = 13$$

$$2x_1 + 2x_2 + 10x_3 = 14$$

5. a) If  $u = e^{xyz} f\left(\frac{xy}{z}\right)$  where,  $f\left(\frac{xy}{z}\right)$  is an arbitrary function of  $\frac{xy}{z}$ , [6]

Prove that  $x \frac{\partial u}{\partial x} + z \frac{\partial u}{\partial z} = y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 2xyz \cdot u$ .

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- b) Prove that  $\sin^5 \theta = \frac{1}{16} (\sin 5\theta - 5\sin 3\theta + 10\sin \theta)$ . [6]
- c) i) Prove that  $\log(\sec x) = \frac{1}{2} x^2 + \frac{1}{12} x^4 + \dots$  [4]
- ii) Expand  $(2x^3 + 7x^2 + x - 1)$  in powers of  $(x - 2)$ . [4]
6. a) Prove that  $\sin^{-1}(\operatorname{cosec} \theta) = \frac{\pi}{2} + i \log\left(\cot \frac{\theta}{2}\right)$ . [6]
- b) Find non-singular matrices P & Q such that  $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$  is [6]  
reduced to normal form. Also find its rank.
- c) Obtain the root of  $x^3 - x - 1 = 0$  by Regula Falsi Method (Take three [8]  
iterations).
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